3.1 Vectors in 3D

Problems Worksheet

$\pi \int_a^b y^2\, dx$

Step Up In Mathematics Specialist Unit 3 and 4

1. Consider the vectors $\boldsymbol{a} = 2\boldsymbol{i} - \boldsymbol{j} + 3\boldsymbol{k}$, $\boldsymbol{b} = \boldsymbol{i} + \boldsymbol{j} - \boldsymbol{k}$ and $\boldsymbol{c} = \boldsymbol{i} + c_2\boldsymbol{j} + c_3\boldsymbol{k}$.

   a. Determine a unit vector in the direction of $\boldsymbol{a}$.

   b. Determine a vector with the magnitude of $\boldsymbol{a}$ and in the direction of $\boldsymbol{b}$.

   c. Determine a relationship between c_2 and c_3 such that $\boldsymbol{a} \perp \boldsymbol{c}$.

   d. Calculate the angle between vectors $\boldsymbol{a}$ and $\boldsymbol{b}$ in degrees to one decimal place.

   e. Determine the vector projection of $\boldsymbol{b}$ onto $\boldsymbol{a}$ and state its meaning.

2. Consider the vectors $\boldsymbol{a} = 6\boldsymbol{i} - \boldsymbol{k}$, $\boldsymbol{b} = \boldsymbol{j} + u\boldsymbol{k}$, $\boldsymbol{c} = v\boldsymbol{i} + 5\boldsymbol{j} - 4\boldsymbol{k}$ and $\boldsymbol{d} = w\boldsymbol{i} + 3\boldsymbol{j} + \boldsymbol{k}$, where u, v and w are scalar quantities. Determine:

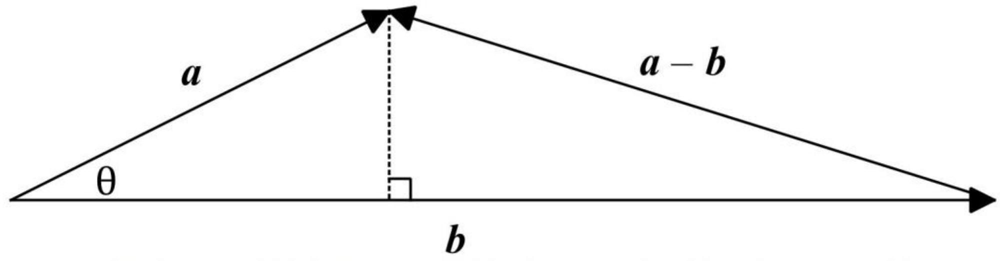
   a. The value(s) of u such that $|\boldsymbol{a}| = |\boldsymbol{b}|$.

   b. The value(s) of v such that $|\boldsymbol{a}| = |\boldsymbol{c}|$.

   c. The value(s) of w such that $\boldsymbol{a} \perp \boldsymbol{d}$.

3. If the points $(-3, -3, 0)$, $(0, n, 3)$ and $(1, 9, 4)$ are collinear, find the value of n.

4. The cosine rule and scalar product. Consider the triangle shown.



a. Using vector notation and with reference to the diagram, prove the cosine rule.

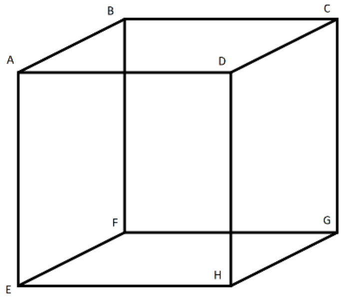
b. Hence show that $\vec{a}.\vec{b} = |\vec{a}||\vec{b}|\cos\theta$.

5. Consider the following proofs in 2D space, using vector techniques.

    a. Prove that the sum of the squares of the diagonals of a parallelogram equals the sum of the squares of its sides.

    b. Prove that the diagonals of a rhombus are perpendicular.

6. Consider the cube $ABCDEFGH$ below. Prove that skew lines CE and FH are perpendicular using vector concepts.



7. Resolve the vector $3\boldsymbol{i} - 2\boldsymbol{j} + 2\boldsymbol{k}$ into two vector components, one that is parallel to the vector $6\boldsymbol{i} - 3\boldsymbol{j} + 2\boldsymbol{k}$.

